Mesh Editing with Curvature Flow Laplacian

Differential coordinates are essentially vectors encoded in the global coordinate system. In differential-coordinates-based mesh editing, they must somehow be transformed to match the desired new orientations, otherwise distortion like shearing and stretching will occur.

We present an iterative Laplacian-based editing framework to solve this transformation problem.

- Positions of the handles are the only required input
- No local frames are required
- Supports point handle editing

Our iterative updating process finds the best orientations of local features, including the orientations at the point handles.

Iterative Updating Framework

- To reduce distortion, the edited mesh should retain:
  - Parameterization information (shapes of triangles)
  - Geometry information (sizes of local features)

- Our system is based on Laplacian coordinates (LCs) $L$ defined on mesh vertex position $V$:
  \[ L = \sum_{j \in C(v)} w_{ij}(v_j - v_i), \]
  In matrix form: $L = I V$.

  We propose:
  - The magnitudes of LCs as geometry information
  - The coefficients of $L$ as parameterization information

  We adopt the cotangent weighting scheme, because this LC approximates the curvature normal.

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  \[ w_{ij} = \cot(\alpha_{ij} + \cot(\beta_{ij}), \]
  \[ L = \sum_{j \in C(v)} w_{ij}(v_j - v_i) = 4 \text{Area}(K_i), \]
  \[ \text{one-ring triangles area} \]
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- The system iteratively improves the vertex positions $V$ and the LCs $L$, minimizing parameterization and geometry distortions progressively.

Reconstruction without Directional Information

Our representation of geometry and parameterization information basically is the decomposition of the global vertex positions into local scalar information.

We can reconstruct the models without using the directions of original LCs: only the magnitudes of the LCs (geometry information) and the Laplacian coefficients (parameterization information) are used.

Application: Spherical Mapping

- Spherical parameterizations using:
  - Mean curvature flow Laplacian
  - Tutte Laplacian

  This is done by setting the curvature field of the LCs to a constant value, and constructing the spherical mapping with our updating method.

Rescaling of LCs

When distances between handles are changed drastically, stretching or squashing distortion occurs. Merely reorientating the LCs cannot produce deformation with small parameterization error.

Our system provides an option to rescale the LCs by the ratio of average edge lengths to reduce such anisotropic scaling.

\[ d_{v_i}^{t+1} = \sqrt{d_{v_i}^t} \left( \frac{\text{Average edge length}}{d_{v_i}^t} \right), \]
where $d_{v_i}^t$ and $d_{v_i}^{t+1}$ are the sums of the triangle areas sharing the vertex $v_i$ in the original mesh and the mesh at time $t + 1$, respectively.

Our system automatically rescales the LCs to eliminate undesired distortion, which is dependent on the geometry complexity and thus it is difficult for user to design a scaling field for the LCs.

Application: Non-shrinking Smoothing

- Smoothed models produced by smoothing the curvature field of the LCs and then reconstructing the models using our updating method.
- The curvature fields of all the examples were smoothed with 10 iterations.